

# Navigation For Low-cost Missions to Small Solar-System Bodies

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## Abstract

A variety of low-cost space missions planned by NASA for flight in the late 1990's and early 2000's will involve rendezvous with, and orbits about, small solar-system bodies such as asteroids and comets. Rendezvous missions of this nature have never been performed, all previous small-body encounters having been flybys. Thus in navigating these missions there are a number of issues and challenges which are new. This paper will identify the different mission phases for small body encounters and the navigation requirements, objectives and goals involved with each phase. In addition, certain practical limitations with respect to mission design will be identified and the scientific information obtained by navigation during the mission discussed.

The phases and issues addressed in the paper are: pre-encounter characterization, encounter and rendezvous with the body, post-encounter characterization, initial orbit strategy and the mission phase itself. Each of these phases have integral importance and are critical to the success of the entire mission. Inherent in each of these phases are all the traditional navigation concerns, such as a *priori* knowledge, maneuver design and execution, data acquisition, orbit determination, orbit reconstruction and control. The paper explains how these traditional roles will be implemented for future small-body missions, including the use of autonomous navigation where practical.

Navigation of spacecraft to and about small solar-system bodies is challenging and raises many issues of fundamental importance which should be understood by the mission navigators, designers and sponsors. This paper will identify the most important issues and discuss ways in which they may be dealt with. It also provides a methodology with which to approach navigation for small-body missions.

## Introduction

Rendezvous missions to small bodies such as asteroids and comets are currently being planned or proposed. The impetus for these ambitious plans are the relatively unknown properties of these members of the solar system. Increased knowledge of these bodies will provide a more

complete picture of the solar system and a better understanding of the processes that formed the solar system.

Navigation of such missions will present challenges never faced in the history of space exploration. The responsibilities and duties of navigation during such a mission are to deliver the spacecraft into a capture orbit at the small body, provide predictions and reconstructions of the spacecraft motion and target the spacecraft into desired orbits during the mission. In order to perform these tasks it is necessary for navigation to have appropriate models of the small body and its force environment. Thus an additional task of navigation is to estimate and construct these models once rendezvous with the body is achieved.

There are critical differences between small body rendezvous missions and the classical planetary rendezvous missions performed in the past and planned for the future. When designing a mission to a planet, there is generally a wealth of information concerning the size, shape, environment and mass of the body, either based on ground observations or on previous encounters. The situation is drastically different with regard to small bodies. Ground based observations cannot provide the same depth of information available for larger bodies. Additionally forces that are small for planetary orbiters may become relatively large as compared to the gravitational attraction of the small body. Finally the shape of the small body tends to be irregular and significantly non-spherical, leading to significant gravitational perturbations which must be estimated before specific orbits can be designed and implemented.

This paper concerns itself only with the rendezvous and orbit phase of a small body mission. Navigation for interplanetary trajectories has been performed often and is relatively well understood (Reference). The paper describes a generic navigation plan for a small body mission and provides justifications where necessary. Numerical values are cited only when appropriate for clarification. The paper is divided into a number of sections, each describing the needs and requirements of the different mission phases. The sections are: pre-encounter characterization, encounter and rendezvous phase, post-encounter characterization, initial orbit and mission phase.

## Pre-encounter Characterization

The comets and asteroids that make up the small bodies of the solar system are so numerous and so diverse that few generalizations can be used to guide a pre-encounter characterization of a target body. Sizes range from a few meters in diameter to over 900 km, densities can range from fluffy cometary structures to solid iron and cometary nuclei may masquerade as asteroids or remain hidden behind a cloud of dust and gas. Rocket-like outgassing thrusts affect the orbital motions of comets and can introduce significant nongravitational accelerations on a neighboring spacecraft. Each target body is an individual and, as a result, each should be intensively studied well in advance of the planned encounter. Carefully planned observing campaigns will yield dividends in terms of the object's ephemeris accuracy at the time of the spacecraft encounter and a first order knowledge of the target object's size, shape, albedo, composition, and in the case of a comet - its outgassing characteristics. This advance knowledge of the target body's characteristics will facilitate the design of science instruments, the sequencing of in-situ observations, the precise location of the target body prior to rendezvous and the optimal design of the spacecraft's orbits about the target body during the rendezvous mission phase.

## Ephemeris Development

The *a priori* accuracy of a target body's ephemeris will depend upon the length of time for which ground-based astrometric observations exist, the accuracy of these data and the proximity of the object to the Earth when the observations were taken. Optical plane-of-sky observations taken during close Earth approaches are very powerful in the orbit determination process. More powerful are radar Doppler and time delay observations, as these data have a far greater fractional precision than traditional optical astrometry (Ostro et al., 1991; Yeomans et al., 1987, 1992). The accuracy of optical astrometric data can be improved by compiling special reference star catalogs in advance and asking experienced observers to reduce their astrometric CCD frames with respect to these reference stars. These extra efforts paid off handsomely for the Galileo spacecraft flybys of asteroid 951 Gaspra on Oct. 29, 1991 and again during the flyby of asteroid 243 Ida on Aug. 28, 1993 (Yeomans et al., 1993, Owen and Yeomans, 1994). From ground-based efforts alone, the relative spacecraft-asteroid position errors were well below 100 km, in the future large format CCDs will be used to capture solar system objects against the highly accurate all-sky Hipparcos catalog. An order of magnitude improvement in ephemeris accuracies may then be possible.

For comets, the ephemeris accuracy also depends upon the ability to successfully model the outgassing, or nongravitational, accelerations that can act upon the comet (Marsden et al., 1973; Yeomans and Chodas, 1989). These cometary nongravitational effects will depend upon a comet's rotation pole orientation and precession, the size and location of the outgassing vents, and the volatility of the vaporizing ices. Unfortunately, only the time-averaged effects of these nongravitational accelerations can be deter-

mined from the use of ground-based astrometry in modeling activities.

## Physical Characterization

Using a combination of visual and infrared photometric techniques and radar observations, meaningful constraints can be placed upon the sizes, albedos, shapes, rotation rates, and rotation pole orientations of many asteroids (Magnusson et al., 1989; Harris and Lupishko, 1989; Millis and Dunham, 1989). In addition, spectral observations of reflected sunlight and radar observations have been used to infer the compositions of many asteroids (Gaffey et al., 1989; Ostro, 1993). However, these techniques rely upon model assumptions and analogies with meteorites so the quantitative characterization of an asteroid's morphology and composition will require a comprehensive set of observations be made from an orbiting spacecraft.

From the orbital analysis of asteroids that perturb one another, the masses of a few asteroids have been determined (Hoffman, 1989). However, for the vast majority of asteroids, there are no mass determinations and only model-dependent estimates of their sizes so that their bulk densities are not known. For comets, the situation is even worse since there are no reliable mass determinations and because their nuclei are often hidden within an atmosphere of dust and gas, there are very few reliable estimates of their sizes. As a rule of thumb, the bulk densities of comets, and the most common C-type, and S-type asteroids are often taken to be about 1, 2.6, and 3.5 grams/cm<sup>3</sup> respectively but the uncertainties on these numbers are a good fraction of the values themselves.

Because the very small bodies of the solar system are probably the result of collision fragmentation in the early solar system, one would expect the smaller objects to be more irregularly shaped. To date, we have images of only the two asteroids (951 Gaspra and 243 Ida) observed by the Galileo spacecraft. Both of these main-belt asteroids are irregularly shaped with the longer axis nearly twice the size of the other two dimensions. The longest axis for Gaspra and Ida are respectively 19 and 55 km. Near-Earth asteroids and comets are likely to be smaller and even more irregular. Two of these objects, 4769 Castalia and 4179 Toutatis, have been "imaged" using radar techniques during very close-Earth approaches (Ostro, 1993). These asteroids are only a few kilometers in size and each appears to have a distinctly bi-modal mass distribution - perhaps a contact binary. This suggests that several small asteroids may be bi-modal in their mass distributions. However, true binary asteroids are likely to be rare. Once placed in orbit about an asteroid, a satellite fragment could remain there over long time periods (Hamilton and Burns, 1992) if not subjected to a major perturbation. However, the difficulty in getting the fragment in orbit initially suggests that satellite fragments would be unusual. Since the relaxation time for a large asteroid to settle down to principal axis rotation is far shorter than its lifetime between collisions, one would expect these asteroids to be in principal axis rotation (Burns and Safronov, 1973). Since this relaxation time is largest for small, irregularly-shaped objects in slow

rotation, a few of the smaller near-Earth asteroids may not have their body axes aligned with the rotation axis.

Of the cometary population, only Halley has been imaged and it too appears irregularly shaped with its longest axis approximately 15 km in extent. When comets are orbiting the sun inside about three astronomical units, their water ice begins to vaporize, releasing the gas and dust particles that comprise the cometary atmosphere. This atmosphere, or coma, effectively hides the cometary nucleus from optical and infrared ground-based observations so that very little can be done to characterize active cometary targets prior to a spacecraft rendezvous. In addition, centimeter-sized dust particles released by the cometary nucleus also makes radar observations difficult even during Earth close approaches. A comprehensive model of the cometary nucleus, including the rotation pole orientation and the location of the outgassing vents, will have to await the arrival of the rendezvous spacecraft. Because this cometary outgassing activity will also introduce nongravitational accelerations upon a rendezvous spacecraft, radio science experiments designed to determine a comet's mass and gravity field from Doppler tracking of the orbiting spacecraft should make the necessary measurements as far as possible from the sun when the comet is least active. Cometary gas production rates, as a function of heliocentric distance, can be estimated using ultraviolet spectral observations of the OH radical and these rates can be used to provide a first-order, a *priori* model of the nongravitational accelerations that might be experienced by a neighboring spacecraft.

For cometary nuclei on the order of a few kilometers in radius, the escape velocity will be a few meters per second while the outflowing gas and dust will travel at velocities of one to two orders of magnitude faster. Thus, dust particles will not remain in orbit about the comet. However, particles larger than a few centimeters would not be completely entrained in the escaping gas flow and might be expected to be in temporary orbits about the nucleus. Torques imparted to the nucleus by the escaping gas and dust should prevent an active cometary nucleus from relaxing to principal axis rotation. In this regard, we note that the analysis of the ground-based and spacecraft data of comet Halley showed this comet to be rotating in a complex manner, not consistent with principal axis rotation (Peale, 1992).

## Encounter and Rendezvous

Upon arrival in the vicinity of the target body the spacecraft is slowed from a body relative hyperbolic velocity to a velocity on the order of meters per second. The spacecraft is simultaneously targeted to a pre-specified flyby radius or rendezvous condition, usually hundreds of kilometers from the target body on the sunlit side. This process must be carried out carefully in order to minimize the delivery errors in velocity and position and the total time of the rendezvous sequence. Initial estimates of the target body mass and size are also acquired.

### Rendezvous Strategy

The rendezvous and injection sequence at small bodies is markedly different than the necessary sequences for orbiter missions to planets. Whereas the injection burn is usually performed at periapsis when flying by a planet, for a small body the ephemeris uncertainties and small mass of the body render this technique useless. Rather, a sequence of slow clown maneuvers must be executed in order to achieve capture at the body.

More than one maneuver is needed to achieve the desired rendezvous speed and altitude because of the expected maneuver execution errors in both magnitude and pointing that result when performing a propulsive maneuver. For example, if the initial hyperbolic excess speed is 1000 m/sec and the desired rendezvous speed at closest approach is 5 m/sec, but the expected spherical execution error is 1/40, then a 10 m/sec spherical error can result. Thus the spacecraft may be moving in an unknown direction at speeds ranging from 0 to 15 m/sec after the maneuver. A series of three to four maneuvers is usually needed, each maneuver being 10% to 50% of the preceding maneuver, until the final speed is achieved. Also, a redetermination of the spacecraft's trajectory after each maneuver is necessary so the next propulsive maneuver can be precisely calculated. By following this plan, the execution error resulting from the last maneuver will be a small fraction of the final desired speed.

The error in the radius of closest approach at the end of this phase is determined by the magnitude of the last maneuver, the maneuver execution pointing error, and the time from the last maneuver to closest approach. The 3- $\sigma$  uncertainty is computed as

$$\Delta r \approx 3\sigma_p T' \Delta V \quad (1)$$

where  $\sigma_p$  is the 1- $\sigma$  execution pointing error of the maneuver in radians,  $\Delta V$  is the delivered maneuver and  $T'$  is the time from the last maneuver to the closest approach. For example, a last maneuver of 5 m/sec,  $\sigma_p = .010$  radians (.57 degrees), and a maneuver time 7 days from closest approach may result in a total flyby error of  $\pm 90$  km. Thus the desired closest approach distance accuracy will determine the size and timing of the final maneuver.

The number of maneuvers needed in the sequence will increase as the size of the maneuver execution errors increase. Large execution errors will also result in more fuel being needed during the rendezvous phase to compensate for these errors, as well as more uncertainty in the timeline of the maneuvers. A rendezvous maneuver sequence which minimizes the total time of this phase would need small maneuver execution errors and would require 3-axis accelerometers on the spacecraft for precise maneuver magnitude and pointing control.

In designing the rendezvous maneuver sequence, trade-offs must be made between the total fuel expended, the size of the maneuver execution errors, the number of maneuvers, the required time between maneuvers, the total time allowed for the rendezvous phase, and the desired accuracy of the state vector at closest approach. Often it is desired to perform the maneuver sequence in as short a time as possible. A reasonable rule-of-thumb for turn-around time

between maneuvers is approximately 7 days. This allows for the reconstruction of the maneuver, re-estimation of the flyby altitude at the target body, design of the next maneuver and sequencing and up-loading of this maneuver. implicit in this design sequence is a re-optimization of the rest of the rendezvous sequence. This turn-around time may be decreased with improved orbit determination, smaller maneuver execution errors or quicker sequence development and implementation.

### Orbit Determination

Orbit determination during the rendezvous sequence is concerned with optical detection of the small body, reconstruction of maneuvers during the rendezvous sequence and in improving the flyby radius uncertainty of the trajectory.

Optical detection of the body is usually the first task to be performed during the interplanetary approach. This is not always possible prior to the initial maneuver, especially for asteroids due to their small size and low albedos. Approach geometries may also affect the detectability of the body, usually due to large phase angles, although restrictions on the pointing of the spacecraft may also interfere. Early detection is desired as it usually improves the relative uncertainty between the spacecraft trajectory and the body ephemeris.

At some range of the spacecraft from the body (dependent on camera properties and the *a priori* ephemeris uncertainties) the combination of the optical image of the body and the accuracy of the image processing will exceed the accuracy of ground based measurements of the body in the plane of the sky (the plane normal to the spacecraft-body look direction). Within this range, images of the body against a star background are used to reduce the flyby radius uncertainties of the trajectory relative to the body. Current image processing techniques can locate images of fully exposed objects whose theoretical apparent size in the image is 1 pixel or less to within a few tenths of a pixel or less. The optical data accuracy should be assessed and compared to *a priori* ground based observation accuracies of the asteroid or comet to determine when the optical data begins to improve the relative uncertainty between the spacecraft and the body. After this point the optical data should be used to supplement the radiometric data. The strength of the optical data continues to increase as the spacecraft approaches the body. During the final days before closest approach appreciable parallax of the body is usually seen which provides trajectory data in three dimensions.

Usual practice is to track the spacecraft frequently following the execution of maneuvers. Even with relatively small execution and pointing errors, it may take two or more days before the maneuver is estimated and the flyby radius uncertainty reduced to pre-maneuver levels. In practice, it is desired to reduce the post-maneuver flyby uncertainty to the level of pre-maneuver uncertainties or less before the next maneuver is executed. This will avoid mis-targeting in the maneuver design due to uncertainties in the relative position of the spacecraft and body. If the

maneuver execution and pointing errors are large, it may take additional time to achieve pre-maneuver confidence in the flyby radius.

The use of accurate on-board accelerometers, designed to precisely control the maneuver magnitude and direction applied to the spacecraft, will decrease the time required to estimate the delivered maneuver. However, there are other factors which place limits on the time between maneuvers. These include the acquisition and processing of optical data and, more significantly, the time to re-design and update the next maneuver sequence. Due to these time delays a practical limit on the time between maneuvers may be 3-4 days at best for fairly large maneuvers. Even then it may be risky to perform more than two maneuvers with such a short time span in between.

Optical and Doppler data are usually sufficient for orbit determination purposes during the rendezvous. Geometries may arise, however, when either the Doppler or optical data may lose one dimension or more of information. Such geometries occur when the spacecraft is traveling in the Earth plane of sky, when there is poor viewing geometry or when there are no stars visible in the camera field of view. In these situations, additional data types are required to successfully perform the orbit determination function.

### Parameter Estimation

During the rendezvous sequence it is possible to begin estimation of the small body parameters, specifically the total mass of the body and the absolute size, or scale, of the body. In some cases preliminary estimates on the second order gravity field of the body may be made and, depending on the camera parameters and body size, mapping of the body surface may be begun. The description of these two additional tasks is given in the next section.

An accurate mass determination of the body may be made by performing a flyby of the small body at a low speed and altitude. However, such an approach is not necessary for an adequate determination of the body mass. During the rendezvous sequence, every time the relative speed between the spacecraft and body is reduced, the ability of the radiometric data, Doppler in particular, to sense the body's mass increases until the flyby radius uncertainty dominates the uncertainty in the mass determination. Thus, optical data also plays a role in the mass estimation, as it provides improving knowledge of the flyby altitude with respect to the body. Given the *a priori* estimate of the body mass, the final segment of the rendezvous trajectory should be designed to allow for the mass to be measured or bounded.

Due to uncertainty in the density of the small body, knowledge of its total mass is not sufficient to determine its size. This information can be deduced geometrically, however, by comparing the change in relative size of the body in the focal plane while tracking the absolute motion of the body. This measurement provides relevant information only during the final week or two of the rendezvous period, assuming that the approach geometry allows for the measurements to be made,

## Initial Characterization

Following rendezvous with the body, the spacecraft is injected into a nominally circular orbit at a pre-determined altitude designed using the *a priori* estimates of the body's size and mass and the properties of the spacecraft imaging system. The duration of this characterization phase is on the order of 10 days, thus the spacecraft will complete much less than one orbit of the body. If the body's image can be resolved during the rendezvous sequence, characterization may begin at that time. Unless the body is grossly different from *a priori* suppositions, the altitude for this phase will not be re-designed during the rendezvous phase.

If time is a priority and the nominal mission does not orbit close to the body early in the mission, the initial orbit may be changed to a slow approach. Then the spacecraft would be targeted to a flyby altitude at the initial orbit altitude, under the restriction that the approach velocity be on the order of 1 m/s or less and that the approach time be on the order of 10 days. These restrictions are to ensure that proper characterization of the body may be performed.

The orbit radius should be sized to allow for estimation of the second order gravity field, at least. If the initial orbit is too close, however, the orbit prediction will be unreliable and have a large uncertainty for a period of time as the filter attempts to solve for the orbit and higher harmonics, lacking any *a priori* knowledge of lower order harmonics.

### Landmark Map and Shape Description

This map of body landmarks and features is used to navigate during the orbit phase and to support science targeting requirements. The optical images provide an initial catalog of landmarks and features and locates them at preliminary levels of accuracy on the body surface. This landmark and feature data is also used to make the initial determination of the spin characteristics of the body (spin rate, orientation, nutation and precession of the body). The expected motion for asteroids will be a near principal axis rotation of the body. For comets, there will in general be sizable nutation and precession of the body, due to the non-gravitational torques the comet nucleus receives from out gassing at every perihelion passage.

Landmark acquisition, selection and processing is complex and data intensive whether done autonomously or with human involvement in the initial map construction. Thus it is preferred to catalog as few landmarks and require as few sightings of landmarks for navigation as is necessary. The density of landmarks needed is a function of the imager field of view, the lowest altitude the spacecraft will achieve and the dependence of the mission on optical data. An alternative to landmark tracking is limb tracking, where the basic estimation process remains the same but it is the limb or shape information of the body that is being estimated and cataloged. This approach will be useful for imagers with a wide field of view, and may actually enable a degree of on-board autonomy as this data type is easier to reduce using simple algorithms.

Previous experience at JPL, with optical navigation picture processing on the ground indicates that 2 to 3 hours is

required to receive, pre-process, review, and extract optical data from an image during the startup phase in which landmarks are being chosen, located, modeled and cataloged. Subsequent images capturing known landmarks can be processed much more quickly (several to many per hour) to improve accuracy and do orbit determination. Down-link transmission time per picture will at most be the time it takes to send one science picture frame. It is usually possible to perform on-board compression of the data contained in an optical picture frame, allowing for a significant decrease in the transmission time of the optical data. See (Gaskell, 1988) for a description of ground processing of landmark data.

Least squares estimators, incorporating dynamic models of the body, are used to improve the estimates of landmark locations and rotation parameters. Initial estimates of rotation and shape can begin as early as first detection by utilizing light curves derived from successive images, identification and cataloging of body referenced latitude and longitude of landmarks can be begun as soon as discerned on the images. Continual updates occur throughout the approach, rendezvous and orbit phases. Surface brightness contours as a function of phase angle and spacecraft/landmark relative geometries are continually updated as more data is acquired.

### Initial Gravity Field Determination

During this phase it becomes possible to reliably estimate the second and third order gravity field of the body. To detect the second order gravity field about a comet requires an orbit altitude of 30 to 50 radii. For determination at an asteroid requires orbit altitudes of 40 to 100 radii. In general, the smaller the body, the smaller the orbit should be. Continuous or near-continuous tracking is usually desired during this phase as this allows for a quick determination of the low-order gravity field. The estimate of this field is initiated using an *a priori* field constructed from the visible shape of the body using constant density assumptions.

The gravitational harmonic coefficients are determined using a least-squares estimator which simultaneously solves for the spacecraft trajectory, body mass and low-order gravity field. The estimation relies on Doppler data to provide measurements of the spacecraft velocity coupled with optical data to provide the body relative position of the spacecraft. The presence of both data types is essential for timely determination as the gravity field must be specified in body fixed coordinates.

The second order gravity field is important as it characterizes the main gravitational perturbations the spacecraft will encounter during the mission. Due to the strength of these perturbations for irregularly shaped bodies, the nominal mission is planned assuming an *a priori* value for these coefficients. Due to the large uncertainties in the *a priori* characterization of the body shape, size and density, these values will be poorly determined. Thus, once the initial model has been improved via estimation by tracking the spacecraft over a 1-2 week period, the nominal mission plan must be re-designed using the new values for the

second and third order gravity field. Thus this phase is crucial for the mission design as the nominal mission plan must be revised and finetuned given the updated model of the body.

#### Inertia Tensor Determination

Given the rotational dynamics and the second order gravity field of the body, it is desired to determine the inertia tensor. Determination of the inertia tensor enables long term prediction of the rotational dynamics and can also shed insight into the internal mass distribution of the body. Given the measured rotational dynamics alone, it is not possible to make a complete determination of the inertia tensor due to an ambiguity in the Euler equations of motion. However, assuming a good determination of the second order gravity field, it is possible to resolve the ambiguity and get a complete solution for the inertia tensor of the body. The process of doing so is detailed in (Miller et. al., 1990). This solution is continually updated as additional data is acquired.

Should the body be very close to principal axis rotation, a complete determination of the inertia tensor cannot be made due to additional ambiguities. However, if this is the case, the propagation of the rotational dynamics becomes simpler and the navigation importance of the complete inertia tensor decreases. Note that no body is in true principal axis rotation and thus at some point during the mission phase the inertia tensor would be determined to some level of accuracy.

#### Initial Orbit

Following the characterization of the body, the spacecraft is transferred to a lower orbit where the higher order coefficients of the gravity field are sensible. The radius of this orbit should be on the order of 7 - 10 radii for a comet and 9 - 15 radii for an asteroid (depending on the size and density of the body). If the mission plan calls for the orbiter to come to within a few radii of the body, this phase is essential for preparing the gravitational model for this event. If the orbiter will not come close to the body, this initial orbit may be targeted to the nominal mission orbit radius. If the target body is an active comet, the outgassing force environment must be characterized during this phase to enable navigation to generate predictions, ensure robustness of the *a priori* navigation plan and to support science desires and goals. The phase is also used to finalize the landmark and shape determination of the body in preparation of the mission phase.

#### Improved Map and Shape Determination

With improving landmark tracking accuracy and increasing familiarity with the body, the basic landmark map and shape determination of the body should be completed during this phase. The relevant coordinate systems, landmark locations and surface characterizations must be communicated to the science and mission design teams for use in specifying desired targets on the surface. Note, however, that if the imager field of view is small, on the order of degrees, it may be necessary to generate additional

landmark maps if the spacecraft transfers to lower altitude orbits. This is required so that there exists a map of the body surface at all relevant resolutions, as the landmark tracking process may easily become ambiguous should the surface area viewed in each frame shrink by an order of magnitude. For a larger field of view imager (on the order of tens of degrees), it is still possible to identify landmarks even with large changes in the orbit altitude.

Surface landmarks should be cataloged at a density such that images containing one or more landmarks can be acquired within a pre-specified time limit. At most this time will be every 2-3 hours. The factors affecting landmark density requirements include constraint policies on camera pointing, camera field of view, camera pointing accuracy, the period and altitude of the spacecraft orbit, the target body rotational dynamics, down-link characteristics and ground processing capabilities. If limb tracking is used to generate optical data, it becomes necessary to determine the degree to which the body shape is to be modeled. This will be a direct function of the desired accuracy of the optical data measurements, with a lower accuracy measurement requiring a lower order model for the body.

The resolution and accuracy of the landmark net and shape characterization will improve throughout the mission. However, real time improvements to the landmark net may not be automatically incorporated once the mission phase begins, in order to avoid confusion from a shifting set of coordinates on the asteroid surface. If improvements to the surface model are made during the mission they will be delivered officially to avoid ambiguity and confusion.

#### Higher Order Gravity Fields

During the initial orbit the higher order harmonic coefficients of the gravity field are determined (4th order and higher). These terms will have a large effect on the short-term dynamics of the spacecraft when within 2 radii of the body, yet they must still be estimated for spacecraft which stay above this limit. Knowledge of the higher order terms reduce errors in the second and third order gravity field determination, allow for prediction capabilities to be extended to a period of days and enables quicker solve times for maneuver execution errors. Again, continuous tracking during gravity field estimation in general enables quicker solution times for the gravity field and smaller uncertainties in the final, determined gravity field. Also, orbits close to the Earth plane of sky (within 5- 10 degrees) should be avoided as the information content of the Doppler data type during such geometries is in general drastically reduced.

Usually, the estimation of the gravity coefficients continues and improves throughout the mission. If, however, these coefficients are determined to within the accuracy constraints needed for the mission, they need not be estimated further during the actual mission phase. Post-mission reconstruction efforts will, in general, estimate them to a higher order of precision.

If the mission plan calls for the spacecraft to descend to lower altitudes (on the order of a few radii) for extended

periods of time, the higher order gravitational coefficients must be estimated prior to descent to the lower altitude. The gravity coefficients are estimated by decreasing the periapsis altitude and tracking during periapsis passage to obtain an enhanced gravity field from an orbit not as affected by the higher gravitational harmonics. In this manner it becomes possible to gradually step into orbits less than two body radii (assuming that the dynamical environment allows for such close orbits).

If the body is a comet, the higher order gravity field will be corrupted by the non-gravitational signatures of the comet outgassing. If the outgassing is not modeled, then the gravitational harmonics may have sizable stochastic components and must continually be re-estimated or estimated as non-zero mean stochastic parameters. If the outgassing is modeled, the corruption of the gravity field may not be as severe, yet will not disappear and will still require longer tracking arcs to converge upon the coefficients.

### Comet Outgassing

If the target body is a comet, the initial orbit is used to estimate the magnitude and variation of the force of the comet outgassing acting on the spacecraft. This includes estimating the effective area to mass ratio of the spacecraft (which will be uncertain), estimating the variation of the outgassing field strength as it varies from the sun-line and as it varies across the body, characterizing the stochastic variation seen in this field and assigning proper correlation times to these stochastic variations.

As the physics of comets are not completely understood, the outgassing pressure at a comet cannot be reliably predicted from *a priori* information. Nonetheless, it is expected that the effective force of the outgassing at an active comet may be as large as 10% of the comet's gravitational attraction. Thus the outgassing force will have a major effect on the spacecraft orbit and must be characterized for both mission design and navigation purposes. Furthermore, this force can act to either decrease or increase the orbit semi-major axis and eccentricity, leading to the possibility of an unplanned spacecraft escape or close fly-by of the nucleus (Scheeres, 1993).

A conservative approach to navigation at a comet would not estimate the outgassing force field and instead view it as a single stochastic parameter, perhaps with some net constant outgassing effect. Such a model would severely limit the prediction capability of the spacecraft motion and would increase the risk of unplanned spacecraft escape or impact. It also requires more frequent tracking of the spacecraft, as the predictive power of the navigation model will be weak and must be continually updated.

A more detailed force model for comet outgassing would contain a few specific items. First, a latitude and longitude map of the larger jets should be made. These locations would be observed from orbit, and their strength estimated indirectly from theory and directly whenever a fly-over occurs. Additionally, some variation law of the comet outgassing away from the sub-solar point (where comet outgassing should be the strongest) would be used to esti-

mate the "global" properties of the outgassing. By proper processing of the gravitational field estimate, the body-fixed component of the outgassing field may be modeled and used in a "secondary" gravity field. This field would be scaled so that the outgassing pressure would vary as any particular point on the comet surface moves towards or away from the sub-solar point.

Such models are relatively simple, yet may allow for a significant improvement in the prediction capability for the spacecraft trajectory and in modeling capability for the design and control of the orbit. The individual terms in these models would be modeled as having a non-zero mean stochastic variation with an appropriate correlation time (or times) to account for the time-varying nature of the outgassing.

## **Nominal Mission Phase**

Navigation during the nominal mission is devoted primarily to the support of the stated scientific objectives of the mission. These objectives require the spacecraft to be targeted to particular orbits and for the navigation team to deliver predictions and reconstructions of the spacecraft position, pointing and associated uncertainties. Additionally, navigation will provide the best current estimates of the gravitational field, rigid body dynamics, inertia tensor, landmark positions, body shape and non-gravitational environment. This section addresses the generic concerns and duties of navigation during the mission phase.

### Orbit Determination

Orbit determination during this phase relies on Doppler and optical data. The Doppler measurements provide direct information on the dynamics of the spacecraft. The optical data provides direct information on the relative location of the spacecraft. Thus these data types are complementary to each other and are usually both essential for a successful mission. There may be cases when, due to a complete, accurate and certain characterization of the small body force environment, the mission may be navigated using only one of these data types. In such situations, optical data is used if the spacecraft uses autonomous navigation, while Doppler data is used if contact with the ground is maintained. For redundancy purposes, both data types are usually kept for the entire mission duration.

During the orbit phase the imager field of view is usually filled completely or substantially by the body. Thus stars will usually not be available to provide an inertial reference for optical images of the body. However, due to the small ranges from the spacecraft to the body landmarks and features during a typical orbit phase (e.g. a few tens of kilometers or less), relatively imprecise knowledge of the spacecraft inertial attitude (e.g. 0.5 - 1 degree) will suffice to locate landmarks and features to within tens of meters on the body surface. This data provides geometrical fixes which, when combined with radiometric data, provide the necessary information to determine the body relative orbits and to perform and plan maneuvers.

It is also possible to rely on shape models of the body alone to generate the optical data. This data type may be preferred for imagers with a larger field of view as it is then possible to image body limbs and terminators without large slew angles of the spacecraft. This data type is simpler to process and transmit in general, and may allow for on-board autonomous data reduction capability. The accuracy of this data type may also be comparable to landmark tracking.

When in orbit, the Doppler data type becomes powerful due to the large variations in dynamical signature within one orbit. This allows for estimation of the orbit and of the force environment about the body. Generally, one 8 hour track of Doppler per day provides good prediction and reconstruction capabilities during periods of lower activity. If the confidence in the force model of the body is high, adequate navigation may be performed with even fewer tracks. Generally, during the mission phase, substantial amounts of science data is generated and transmitted to the ground. Since Doppler data may be acquired with no signal degradation whenever a link between the spacecraft and Earth is established, the tracking needed for returning the science information often provides navigation with sufficient amounts of tracking information.

### Orbit Prediction and Reconstruction

The ability to predict and reconstruct the spacecraft trajectory about the small body is a crucial service required by the science team. The ability to predict trajectories in advance allows the science team to consciously choose target orbits and know in advance the expected deviation from these orbits, allowing for robust sequences to be designed for instrument measurements. The ability to reconstruct the trajectory is used after the primary mission is completed, when there is time to perform a detailed and precise analysis of the results. Then it is desired to know the spacecraft position and pointing relative to the body at the epochs when measurements were made.

The ability for navigation to predict and reconstruct orbits is usually limited by the body model and the information content of the navigation measurements. For asteroids, the body model may be improved to high levels of resolution if tracked over long time spans and will be limited by the knowledge of the gravitational harmonics, solar pressure model and the inherent accuracies of the radiometric and optical measurements. If maneuvers are performed frequently, the ability to predict is limited to the expected execution errors and the ability to reconstruct is limited to the ability to estimate the maneuvers. These considerations drive the need for frequent tracking following maneuvers.

For comets, there will be a fundamental limit on the predictability of the orbits, due to the stochastic nature of the outgassing forces. The size of this limiting effect will be a function of several items. If a modeling capability of the outgassing is used, then prediction times may be extended significantly. Conversely, if the outgassing is not modeled, the predictive ability of navigation may be severely limited, probably to the order of the correlation time of the

stochastic outgassing effect, which may be on the order of hours or days. If tracking is dense enough, the ability to reconstruct orbits about a comet may be fairly strong, as the analyst may use stochastic parameters to correlate the motion between tracking passes. This assumes that the tracking is performed more frequently than the correlation time of the outgassing. Should this not be the case, the ability to reconstruct will begin to degrade.

### Orbit Control and Stability

An orbiter at a small body will, in general, encounter non-Keplerian forces of a much larger relative magnitude than planetary orbiters would encounter. The main perturbations the spacecraft must contend with are the effects of an irregular body shape on the gravity field, solar radiation pressure, comet outgassing and the solar tide, in comparing the absolute magnitudes of these effects, the solar tide does not play an appreciable role over short time periods, except for larger orbits about larger bodies, such as large main belt asteroids. For large asteroids, the major effects are due to the non-spherical shape of the body. For small asteroids, the solar radiation pressure may become an important force. For comets, the solar radiation pressure and the comet outgassing tend to be the major forces to contend with. If a low altitude orbit is achieved at a comet, the shape effects must also be considered.

The effect of the body's irregular shape on the spacecraft orbit may be quite severe. For smaller, and hence more irregularly shaped, asteroids the shape may cause radial instability in the spacecraft orbit, leading to a crash on the asteroid within a short time period (Scheeres, 1994a). This instability occurs when the spacecraft is in a near-synchronous orbit about the body. Thus, low altitude orbiters at asteroids must generally follow retrograde orbits in order to eliminate these instability problems.

In addition to potential instability problems, the effective oblateness of an asteroid will be a significant effect. This effect is most easily characterized by the " $J_2$ " term, although it can be more accurately characterized by considering an oblate spheroid model (Broucke and Scheeres, 1994). The value of the  $J_2$  parameter for asteroids are expected to range up to 0.1 for a maximum value. This leads to precession of the orbit node and argument of periapsis at rates up to 45 degrees/day for a 2 radii orbit at a 45 degree inclination to the mean asteroid rotation pole. Control of this precession about larger asteroids is not feasible due to the relatively large maneuver cost and frequency associated with this precession rate. About smaller bodies it may become possible to control the orbit plane with respect to this precession, although the cost of this control may be on the order of 100 m/s over the entire mission. As an example, the average cost to maintain a preferred inertial orbit orientation about an asteroid is, approximately,

$$\Delta V = \frac{3\mu J_2 |\sin 2i|}{4a^4} \quad (2)$$

where  $\Delta V$  is the necessary maneuver magnitude needed per unit time,  $\mu$  is the gravitational constant of the asteroid,  $J_2$  is its oblateness term,  $i$  is the orbit inclination



with respect to the mean asteroid rotation pole and  $cc$  is the radius of the circular orbit about the asteroid.

If the asteroid is small, and the orbit altitude further than a few radii away from the body, the solar radiation pressure forces begin to dominate the spacecraft dynamics. Orbits may be designed which effectively null out the effects of the solar radiation pressure, however these may not be orbits of interest to the science team. When not in such orbits, the effect of this force is to increase the eccentricity of the orbit towards unity, as well as change the orbit inclination, node and periastron while the semi-major axis remains constant on average. Due to the increasing eccentricity, occasional corrective maneuvers must be performed to restore the orbit. The frequency of such corrections will vary depending on the size and location of the asteroid, but may be as high as one maneuver per week to control the eccentricity alone. Sec (Scheeres, 1994b) for a description of satellite dynamics at an asteroid.

If the body is a comet, its total mass will usually be small and hence both the solar radiation pressure and the comet outgassing will have large effects on the spacecraft orbit. Reference (Scheeres 1993) discusses the general dynamics of a spacecraft about a comet, assuming a simple form for the outgassing pressure. In general, the semi-major axis and the eccentricity of the orbit will have secular drifts, due to the combination of the forces. Corrections must be performed occasionally to correct the orbit back to its desired state. Based on simple models of outgassing, it is possible to design orbits which remain stationary about the comet, although these orbits may not be of scientific interest. Should the spacecraft fly over an active outgassing jet, there may be a large dynamical effect on the spacecraft trajectory. If such fly-throughs should be anticipated, and the spacecraft tracked subsequent to such an event, to enable correction for any large perturbation in the orbit semi-major axis or eccentricity.

Propulsive maneuvers are required from time to time to control or alter the orbit. Because of maneuver execution errors, uncertainty in the spacecraft's position and velocity will exist for a time after the maneuver. These uncertainties are reduced and execution errors detected once the orbit is redetermined using Doppler and optical measurements, which usually requires a day or so of tracking. Note that within a few days after the maneuver, the spacecraft may be many kilometers off in its predicted down-track position due to maneuver execution error. Yet once the orbit is redetermined and the maneuver determined, accurate predictions of the trajectory can be made and the instrument sequences redesigned as appropriate.

#### Autonomous Control of Spacecraft

Major mission costs are often incurred in maintaining large flight teams for sequencing, navigation, attitude control and other essential mission tasks. Autonomous navigation of a spacecraft has the potential for reducing the size of ground teams needed to plan and process the radio-metric and optical data and to perform maneuvers.

Various levels of autonomous control are possible. In the simplest case, the spacecraft is autonomously controlled to

nadir track the small body. This would relieve the ground from needing accurate spacecraft down-track predictions for the design and sequencing of spacecraft pointing. The simplest implementation of this control would require a large field of view imager, not necessarily very accurate, that would maximize the image brightness and hence point the spacecraft to nadir. A more sophisticated approach would have the spacecraft estimate the centroid of the body, using an elementary on-board model of the body and occasional limb scans. No record of the data need be kept on-board, although some of the data should be returned to earth for use in orbit determination. The ground would still perform the orbit determination and navigation tasks, but the need for accurate pointing predictions could be eliminated.

No attempt is made on-board to improve the orbit knowledge, the nadir pointing is simply readjusted. The science data is limited to viewing in the nadir direction. Situations in which there is no signal would be treated by making no adjustment for a period of time known to the algorithm. Loss of imaging signal for longer times would necessitate a call to earth.

A more accurate version of the approach would require a low order model of the asteroid and some information on the location of the sun and the spacecraft trajectory about the object. Using this data the spacecraft instruments could be pointed autonomously to pre-programmed surface locations. To process the optical data the on-board computer would either recompute scenes or store the expected scenes in memory. All these scenes would be low order to minimize memory and computation costs.

If the science measurements are made when large spacecraft trajectory perturbations occur the above scenario may still work, but for shorter periods of time. Then, in addition to tracking the nadir, the wide field imaging data could also be exploited to estimate the spacecraft position in inertial space. The spacecraft computer would have the necessary models to recompute low order scenes in the wide field imager for comparison with the actual data. The residuals would indicate the deviation of the current position from the expected position, allowing the spacecraft to update its current position estimate.

Given moderately accurate attitude control knowledge (on the order of 0.5 degrees), and a low order model for an asteroid, it would be possible to autonomously estimate the position of the spacecraft to the order of 500 meters in a low asteroid orbit. Position fixes taken around the orbit (at least six per orbit) are used to continually update the orbit position and compute the confidence in the position estimate. If the dynamical perturbations grow large enough, the spacecraft would notify Earth and a ground-calculated maneuver would be radioed to the spacecraft. Future autonomous navigation would use the on-board orbit estimate to compute and execute one of a restricted suite of trajectory correction maneuvers.

#### Navigation Products as Science

In the process of navigating the spacecraft around the small body, there are a number of navigation products

which have a scientific interest beyond their navigation use. These items include estimates of the body gravitational field, density, surface map and shape, inertia tensor, rotational dynamics and outgassing field of a comet. The estimates of these items are continually updated as tracking and optical data are reduced. Models used for orbit determination contain all these items as estimation parameters in order to generate accurate body relative coordinates and to enable the orbit trajectory to be predicted and reconstructed.

These items are usually classified under the heading of radiometric and imaging science as they are determined by reducing the Doppler and optical data acquired during the orbital phase of the mission. If this data is archived, then more precise estimation of these parameters is performed during the reconstruction of the spacecraft trajectory, when these items are estimated using the totality of data available to the analyst. The procedure for gaining a final, best estimate on these quantities is generally the same procedure as was used to generate orbit determination during the mission phase, although all the tracking data is now combined into one effective data arc (Konopliv, 1993).

Another navigation product which is used for scientific purposes is the improvement of the ephemeris of the small body. By tracking the satellite in orbit about the small body, the heliocentric trajectory of the body is also tracked. Thus, substantial improvement in the ephemeris is enabled by tracking the spacecraft over the length of the mission phase. Generally, Doppler data is sufficient to provide marked improvements to the small body ephemeris, although occasional ranging data allows for even further improvement in the body ephemeris. As the range data is not explicitly needed for navigation during the orbital phase, this data type is often considered to be a science measurement once the orbital phase of operations has begun.

## conclusion

Described in this paper is a complete and generic plan for navigating a spacecraft to a small body. The plan is outlined in five phases: pre-encounter characterization, encounter and rendezvous, initial characterization, initial orbit and nominal mission phase. Each section describes the general tasks performed by navigation for mission support and the basic requirements that navigation will need to carry out these tasks. By focusing on the minimum requirements, the paper describes the essential tasks that must be retained for a low-cost mission. A discussion on autonomy indicates how the ground support system may be reduced during the nominal mission phase. Missions to small solar system bodies are challenging and will require navigation to develop new tools and techniques to face these unique situations. In a period of tightening space exploration budgets, it is crucial to identify the essential tasks for any mission, so appropriate budgeting and effort can be expended for the resolution of these tasks.

## Acknowledgements

The research described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

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